**Phys 410**
**Spring 2013**
**Lecture #34 Summary**
**17 April, 2013**

We started with Euler's equations for a rotating object subjected to a net external torque $\vec{\Gamma}$. The equation of motion as witnessed in the body frame is $\vec{\Gamma} = \left(\frac{d\vec{L}}{dt}\right)_{Body} + \vec{\omega} \times \vec{L}$, which translates in component form into the Euler equations:

$$\Gamma_1 = \lambda_1 \dot{\omega}_1 - \omega_2 \omega_3 (\lambda_2 - \lambda_3)$$

$$\Gamma_2 = \lambda_2 \dot{\omega}_2 - \omega_1 \omega_3 (\lambda_3 - \lambda_1)$$

$$\Gamma_3 = \lambda_3 \dot{\omega}_3 - \omega_1 \omega_2 (\lambda_1 - \lambda_2)$$

We considered the special case of zero torque $\Gamma_1 = \Gamma_2 = \Gamma_3 = 0$. If an object starts off rotating solely about one of the three principal axes, these equations say that it will stay rotating about that axis forever (as long as the net external torque remains zero). These equations also show that if the object is started into rotation along any two of the principal axes, it will soon develop a component of rotation about the third axis – hence it will wobble.

The third application of these equations was to the case of an object rotating about one principal axis ($\hat{e}_3$), but then given a small kick to produce rotations about the other axes ($\hat{e}_1$). The analysis led to a simple equation of motion: $\ddot{\omega}_1 = -\left[\frac{(\lambda_3-\lambda_2)(\lambda_3-\lambda_1)}{\lambda_1\lambda_2}\omega_3^2\right]\omega_1$. This yields simple-harmonic motion (SHM) if $\lambda_3$ is either the largest or smallest of the three principal moments. SHM means that the motion about the original axis is stable. If $\lambda_3$ is the middle eigenvalue, then the square-bracket term is negative, yielding a solution for $\omega_1(t)$ that grows exponentially in time. This is a sign of instability. We demonstrated this phenomenon with a book, where motion around the principal axes with largest and smallest moments was fairly stable, while motion about the third (middle moment of inertia) was clearly much less stable.

We then began a discussion of coupled oscillators by considering two masses on a friction-less surface, with 3 springs between them. The left mass ($m_1$) is connected to the left wall by a spring of spring constant $k_1$, while the other mass ($m_2$) is connected to the right wall by a spring of spring constant $k_3$. The two masses are also directly connected to each other by a third spring characterized by $k_2$. In the absence of spring 2, the two masses would oscillate independently at their own natural frequencies. However, with the coupling, they will have a new type of motion characterized by 'normal modes.'